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A Study of Shear Angle Relationships in Shearing Process on the Shear Plane and the Rake Face in Orthogonal Cutting

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A new shear angle relationships was analysed with reference to a shear friction factor m in shearing processes on the shear plane and the tool rake face. The theoretical formula are shown to be in good agreemeent with experimental cutting data. The currently developed theoretical equations represent further generalized cutting process. It is found that Lee and shaffer's equation is a special solution of the presented equations. The shear friction factor m shows a clear description of machining characteristics of metals. It was also found that the factor m establishs a new shear angle relationships together with cutting conditions and tool geometry.

Key words : Shear Angle, Shear Process, Friction Rrocess Chip Formation, Shear Friction Factor

1. Introduction

In the theoretical study of metal cutting. hundreds of papers have been published in the past several decades. However, due to the difficulty of theoretical prediction of the shear angle relationships, the metal cutting theory still far beyond to predict accurate cutting forces. Thus, many researchers have concentrated their efforts to study metal cutting for a better understanding of the mechanism of chip formation which dominates the shear angle but, until the present, no rule that can be used for the prediction of shear angle relationships has been found(Arai, 1976).

In general, none of proposed theories of orthogonal cutting is in quantitatively agrees with all of the experimental observations even for a limited range of cutting conditions(Shaw, 1953, Dewhurst, 1978).

In view of the concept that the functional relationships between the shearing process and the friction process seemed to be a determining condition for the solution. Shaw et al.(1953). introduced the variable η' in order to indicate the possible dependency of the two processes. The constant K introduced by Merchant(1945) assumes that the shear stress on the shear plane is a linear function of the normal stress and it may also be interpreted as a measure of the interrelationships of two processes. This assumption, however, does not appear to be supported by experiment, is contrary to the general concepts of the plastic deformation. Thomsen et al.(1959) also introduced a parameter as an indication of the dependency of two processes. namely. the shearing process on the shear plane and the friction process on the rake face of the tool. The parameter, designated by η, was defined by the ratio of the possible minimum energy input for cutting to the actual energy input for the same friction angle and rake angle. The factor, η, was assumed an indication of the interrelatonship between the shearing process on the shear plane and the friction process on the rake face of tool without giving the detailed consideration to the interrelationships itself.

Noting the significance of the deqendency of shearing and friction processes on the steady state configuration. a new parameter called shear friction factor m which is the ratio of the shearing stresses on the shearing plane and tool face is

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proposed in the development of the analysis in order to bring this dependency into the theory. The introdution of this parameter m results in a clearer understanding of the cutting process.

2. The Proposed Cutting Model

In an attempt to construct a complete solution for the metal cutting problem, one must examine the possible stress fields in the workpiece as well as those in the chip. In the theories discussed earlier. the stress distributions were considered only for a part of the body. The force balance method to be discussed subsequently, however. permits one to examine those solutions more closely.

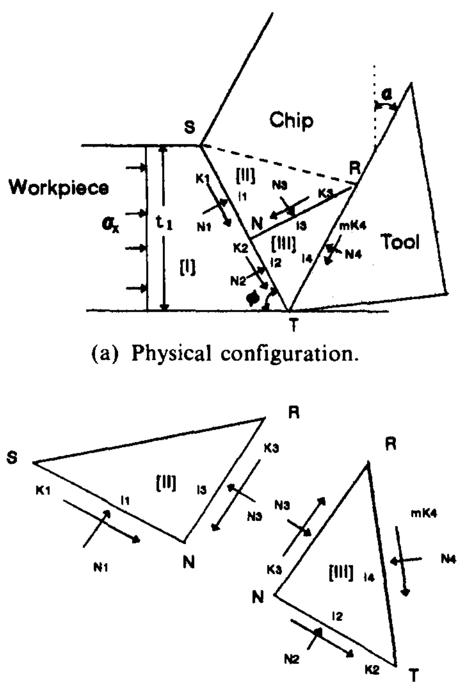
Westwood and Wallace(1960) have introduced the method of the force balance as an upper-bound value for the load, and calculated the upper-bound loads for several metal forming processes by using the Coloumb friction forces on die material interface, comparing their results with the upper-bound solutions from kinematically admissible velocity field of the same configuration. showing the identity between the two approaches. Grzymkowsky and Mroz (1972) also have treated seberal problems by force balance method which account for the upper-bound solution of kinematically admissible velocity field. Avizur and Choi (1986) concluded that two upper bound solutions, by the force balance method and by kinematically admissible velocity fields, always give idkentical results when the geometrical configuration of the deformation model remains the same by detailed calculations for strip drawing and for the friction model.

The theory to be used assumes an ideally plastic material which does not have work hardening. This is known to be a good apprximation for iron and steels at high strain rates which occur in machining(Lee, 1951). Thus we can adopt the rigid plastic type of theory in which materials subjected to strains of elastic order only is considered rigid. The stress must be satisfied the equilibrium equations, because inertia forces are negligible, and the yield con dition in the region of plastic flow. The velocity field of flow must determine strain rates which satisfy the Mises flow condition. Let us consider the application of the theory which is similar to Lee-Shaffer model.

In their slip line solution. Lee and Shaffer considered the state of stress of the chip through which the forces exerted by the tool were transmitted to the shear plane. The shear plane is a line in a two dimensional cut along which the tangential component of velocity is continuous. Lee and Shaffer to apply the slip line theory of perfectly plastic solids to the problem of metal cutting assumed that a certain slip line field (plastic zone) exists within the chip which is composed of two families of parallel straight lines.

The stresses are assumed to be uniform throughout this zone, which is bounded by the shear plane, the tool. and the imaginary boundary across which no stresses are transmitted. Under these conditions, the same deformation model was adopted here for the balance of forces as shown in Fig. 1.

In triangular STR of Fig. 1(a), the metal is



(a) Physical configuration.

(b) Acting fonces

Fig. 1 Forces system acting in orthogonal metal cutting moving as a rigid body parallel to the cutting velocity. The incoming the work material moves axially as a rigid body with a cutting velocity Vc. The shear and normal stresses on the surfaces l_1, l_2, l_3, and l_4 are considered. The force balance in orthogonal cutting is shown in Fig. 1(b). In zone I, the total horizontal and vertical forces are zero as shown by Eps. (1) and (2) respectively.

$$\sigma_x t_1 - N_1 l_1 \sin\phi - k_1 l_1 \cos\phi - N_2 l_2 \sin\phi - k_2 l_2 \cos\phi = 0 \quad (1)$$

$$-N_1 l_1 \cos\phi + k_1 l_1 \sin\phi - N_2 l_2 \cos\phi + k_2 l_2 \sin\phi = 0 \quad (2)$$

As has been recognized widely. the shear stress on the shear plane is practically constant for each steel in a fairly wide range of cutting conditions. From Eqs. (1) and (2). we obtain Eq. (3) if all $k_i = k$

$$\sigma_x = \frac{2k}{\sin 2\phi} \quad (3)$$

Where N_1, N_2 and k_1, k_2 are the normal and stresses on l_1 and l_2 respectively, and ϕ is the shear angle and t_1 is the depth of cut.

In zone II. The total total horizontal and vertical forces are zero respectively. Thus

$$N_1 l_1 \sin\phi + k_1 l_1 \cos\phi - N_3 l_1 \cos\phi - k_3 l_1 \sin\phi = 0 \quad (4)$$

$$N_1 l_1 \cos\phi - k_1 l_1 \sin\phi + N_3 l_1 \sin\phi - k_3 l_1 \cos\phi = 0 \quad (5)$$

From Eqs. (4) and (5). we obtain

$$N_1 = N_3 = k \quad (6)$$

and NNk and k are normal and shear stresses on l_1 and l_3 respectively.

By substituting Eq. (6) into Eq. (2) we obtain

$$N_2 = \frac{k\{l_1(\sin\phi - \cos\phi) + l_2 \sin\}}{l_2 \cos\phi} \quad (7)$$

In zone III. The total horizontal and vertical forces are zero respectively. Thus

$$N_2 l_2 \sin\phi + k_2 l_2 \cos\phi + N_3 l_1 \cos\phi + k_3 l_1 \sin\phi - N_4 l_4 \cos\gamma - m k_4 l_4 \sin\gamma = 0 \quad (8)$$

$$N_2 l_2 \cos\phi - k_2 l_2 \sin\phi - N_3 l_1 \sin\phi + k_3 l_1 \cos\phi + N_4 l_4 \sin\gamma - m k_4 l_4 \cos\gamma = 0 \quad (9)$$

The following length of the each region is derived from the geometrical condition in the developed cutting model.

$$l_1 = \frac{t_1 \cos(\phi-\gamma)}{\sin\phi\ \{\sin(\phi-\gamma) + \cos(\phi-\gamma)\}} \quad (10)$$

$$l_2 = \frac{t_1 \sin(\phi-\gamma)}{\sin\phi\ \{\sin(\phi-\gamma) + \cos(\phi-\gamma)\}} \quad (11)$$

$$l_4 = \frac{t_1}{\sin\phi\ \{\sin(\phi-\gamma) + \cos(\phi-\gamma)\}} \quad (12)$$

Where t_1 is the depth of cut, γ is the rake angle, ane l_4 is the chip-tool contact length on the rake face. From Eqs. (8)~(12). we obtain

$$N_1 = \frac{k\{m \sin(\phi-\gamma) + \sin(\phi-\gamma) + \cos(\phi-\gamma)\}}{\cos(\phi-\gamma)} \quad (13)$$

and N_2. N_3. N_4. k_2. k_3. and mk_4 are normal and shear stresses on l_2 l_3 and l_4 respectively.

From Eq. (1) to Fq. (9) equations form a set of simultaneously linear equation which is constituted of unknown variables σ_x , N_i, k_i and m.

Wher m is determined by the following equations and it is dependent on the friction process on the rake face of the tool.

$$m = \tau_t / k \quad (14)$$

Where τ_t is the shear friction stress and k is the maximum shear strength.

Thus the shear friction factor m indicates the degree of the friction stress necessary for cutting relative to the maximun shear strength of the material on the rake face, if k is equal to the shear stress on the shear plane τ_s, m is expressed as follows.

$$m = \tau_t / \tau_s \quad (15)$$

The angle ρ is the friction angle $\tan^{-1}(F/N)$ together with ϕ yields equation.

$$\tan\rho = \frac{km}{N_4} = \frac{m}{m \tan(\phi-\gamma) + \tan(\phi-\gamma) + 1} \quad (16)$$

Solving Eq. (16) for only ϕ. we obtain

$$\phi = \tan^{-1}\frac{m \tan - \rho}{(1+m)\tan\rho} + \gamma \quad (17)$$

Equation (17) can be rewritten in terms of the friction of coeffiicient on the rake face.

$$\phi = \tan^{-1}\frac{m - \mu}{(1+m)\mu} + \gamma \quad (18)$$

Where is the coefficient of friction on the rake face.

From Eq. (16). We obtain the tangential linear equation in a point as follows.

$$\phi-\left(\tan^{-1}\frac{1-\tan\rho_0}{2\tan\rho_0}+\gamma\right)$$
$$=\frac{-2\sec^2\rho_0}{5\tan^2\rho_0-2\tan\rho_0+1}(\rho-\rho_0) \quad (19)$$

From the Lee-Shaffer equation the value of slope is -1. As a result, the friction angle ρ is $\pi/4$ and the thagential line contacts at point $(\pi/4-\gamma.\gamma)$ with the given curve when $m=1$. Substituting $\rho_0=\pi/4$ into Eq. (19), the equation is equal to the Lee-Shaffer's Eq. (20) as follows.

$$\phi=-\rho+\gamma+\pi/4 \quad (20)$$

Therefore Lee-Shaffer's equation is a special solution of the derived Eq. (16) such as a tangential equation when m is 1.

3. Shear Friction Fraction Factor m in Shearing Process on the Shear Plane and Rake Face

The deformation process on the shear plane takes place under unusual conditions; these are, high normal pressures on the shear plane. extremely high strain rates, large finite strains, and high temperatures. These are reasons why it is difficult to establish a relationships in metal cutting between the plastic flow properties of the material and its plasticity conditions.

Experimental investigations(Kobayashi, 1959) have confimed that the shearing stress on thesr remains essentially constant for a given material over a wide range of cutting conditions. Typical data points(Eggleston, 1959) for a SAE1113 obtained for a wide range of cutting conditions are shown in Fig. 2.

It may be of interest to note that the average shear on the shear plane simply remains constant, and it permits the introduction of shear friction factor m into theory as has bee shown repeatedly.

Shear stresses on the rake face are also plotted an functions of feed in Fig. 3 for the various rake angles.

It is to be noted that these stresses are essentially constant under given test conditions. The shear friction factor m indicates the ration of the shear stress on the shear plane to the shear stress on the rake face. In introducing m. it is assumed that this factor is an indication of the interrelationships between the shearing process on the shear plane and the friction process on the rake face of the tool.

Figure 4 shows the effect of rake angle γ on shear friction factor m for the SAE 1113 where the variation of m is in the range of approximately 0.55~0.65.

Figure 5 also shows the effect of rake angle γ on shear friction factor m. It is noted that the variation of m is very small. Since shear stresses on shear plane and rake face of tool are essentially costant, shear friction factor m is also

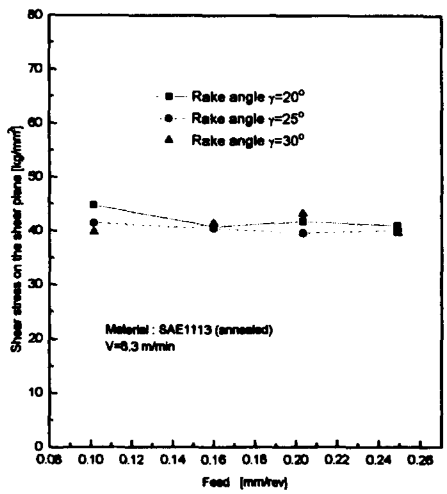


Fig. 2 Correlation between feed and shear stress on the shear plane quoted from Eggleston(1959)

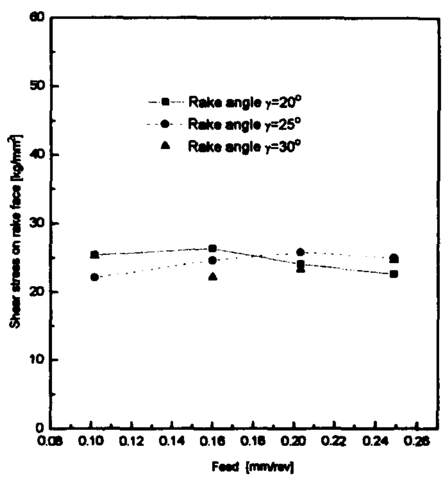


Fig. 3 Correlation between feed and shear stress on the rake face quoted from Egglestion(1959)

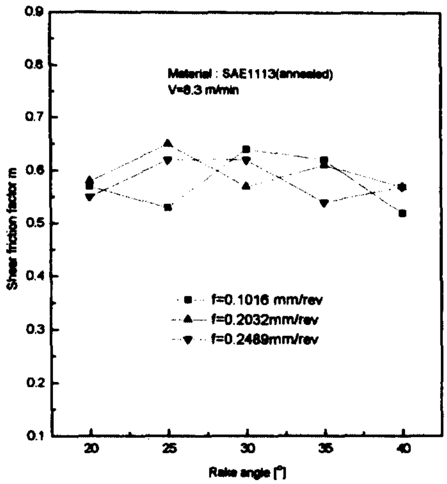


Fig. 4 Effect of rake angle on shear friction factor m

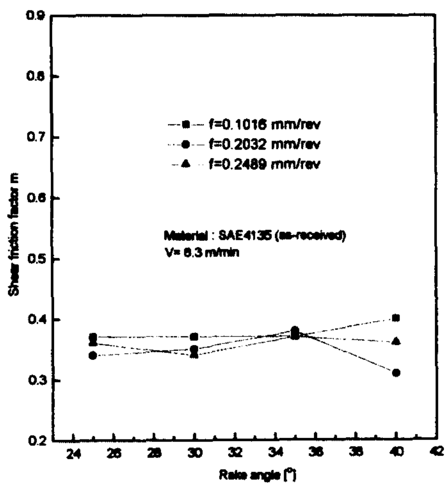


Fig. 5 Effect of rake angle on shear friction factor m

costant in metal cutting.

4. Chip-Tool Contact Length

The prediction of the shear angle relationships is that for a given tool rake angle, with a fixed value of m. a range of deformation modes are possible with varying amounts of chip curvature.

The Lee-Shaffer solution is the mode of deformation with minimum chip thicknessm mininum contact length between the tool and work material(Dewhurst, 1979). If the chip-tool contact length is controlled, then the result is to pick up one solution from the possible solution range. Moreover, if the rake face of the cutting tool is gradually decreased in length, then a gradual reduction in chip curvature would be expected until the Lee-Sharrer solution reachs the given straight chip formation.

Figure 6 is experimental results proposed by Dewhurst and it supports Lee-Shaffer's cutting model. For cutting tests with steels, brass and aluminium, Dewhurst reported, that with a 20° rake angle tool, reducing the tool contact length always results a decreasing chip curvature until the chip straightened completely.

If the contact length between the tool and chip is controlled, then the result clearly picks up one solution.

Figure 7 shows a function of chip-tool contact length versus feed for SAE 113 material.(Eggleston,1959)

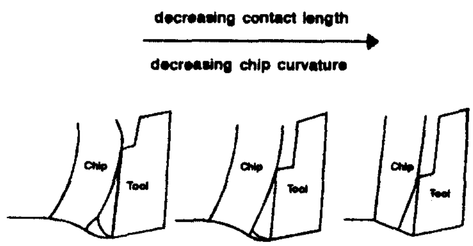


Fig. 6 Cutting with restricted face tool; reducing the contact length eventually gives the Lee-Shaffer solution

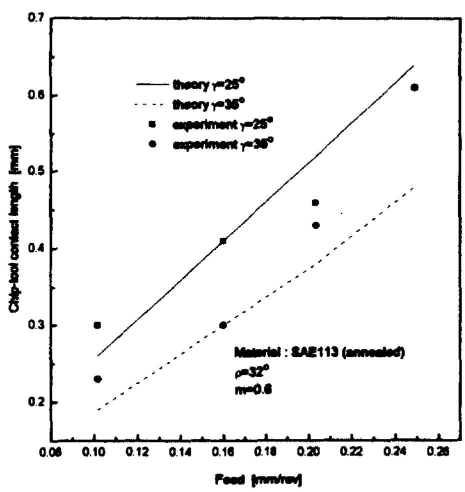


Fig. 7 Effect of feed on chip-tool contact length

It is noted that experimental points are almost plotted over predicted line derived from Eq. (12).

5. Discussion

From a consideration of these limited results, it can be concluded that the m of Eqs. (17), (18) are not function of γ or feed but only slightly dependent on cutting fluid. The reason why m is not equal to 1. as might be expected at first, is undoubtedly due to the fact that the shear process on the rake face of tool is carried out in the vicinity of the hard surface of the tool under complex stress conditions and with imperfect welding. In addition. the metal on the rake face possesses is higher temperture than on the shear plane. Hence one should expect to see the shear friction m rise as increasing the cutting velocity. Figure 8 shows the agreement between theory from Eq. (17) and experiment(Eggleston, 1959) for various feeds. There is a little difference between theory and experiment for incrasing rake angle. There is a little defference between theory and experiment for increasing rake angle. Equation (17) is plotted in Figs. 9, 10 on the graph of ϕ versus $(\rho-\gamma)$ with shear friction factor m for rake angle 10°, 20° respectively.

It may be noted foom the Fig. 9, Fig. 10 that Lee-Shaffer's straight line is tangential line of the derived curve when $m=l$.

Figure 11, Fig.12 shows the calculated values of shear angle ϕ plotted against the $(\rho-\gamma)$, for various values of rake angle γ with Merchant and Lee-Shaffer cutting equations. In these figures the analytical values of ϕ were computed by using the proposed Eq. (17). Figure 11 shows the agreement between theory and experiment for the rake angle of the tool of 20° with curves correspon

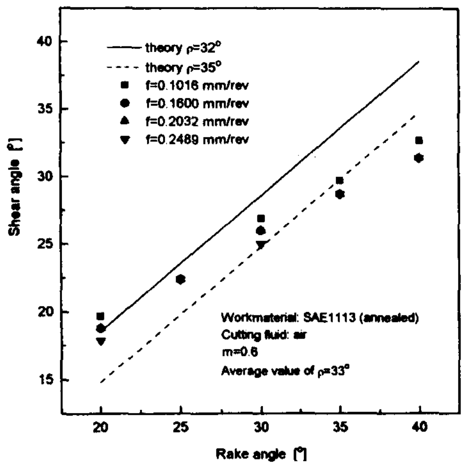


Fig. 8 Correlation between rake angle and shear angle

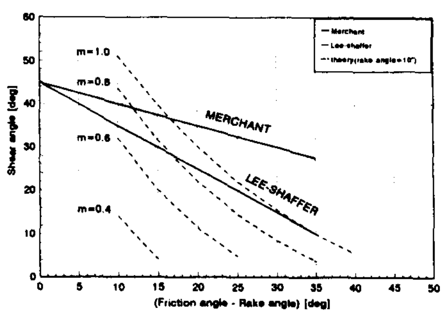


Fig. 9 Comparision of shear angle relationships with shear friction factor m

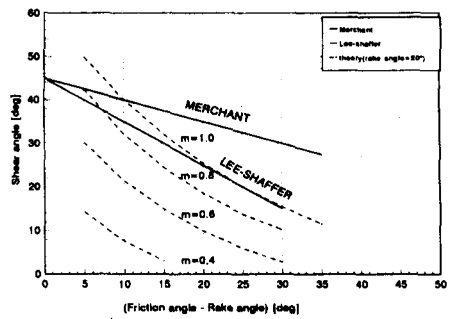


Fig. 10 Comparision of shear angle relationships with shear friction factor m

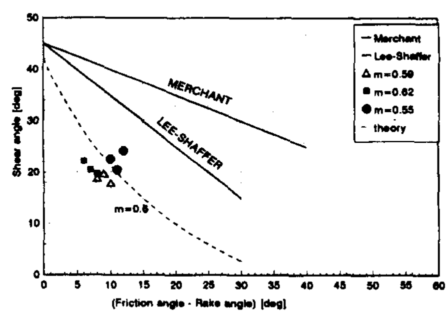


Fig. 11 Comparision of shear angle relations to experimental data by Eggleston(1959)
Work material : SAE 1113, f=0.016−0. 2489mm/rev, V=8.3m/min, rake angle=20 deg

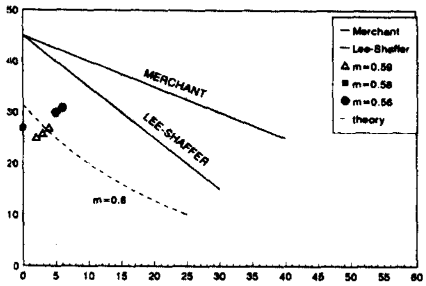


Fig. 12 Comparision of shear angle relations to experimental data by Eggleston(1959) Work material : SAE 1113, f=0.016−0.2489mm/rev, V=8.3m/min, rake angle=20 deg

ding to specified values of $m=0.6$. From the results, theoretical values well predict the experimental ones. The shear friction factor m rises with increasing rake angle. Summarizing the results we can conclude the agreement between the theoretical curves and the experimental results(Eggleston. 1959) for shear angle relationships with shear friction factor is fair.

6. Conclusion

A new analysis to shear angle relationships in shearing process on the shear plane and on the rake face in orthogonal cutting was carried out and the results were compared with available data. The important results and conclusion are summarized below.

(1) Shear friction factor m permits a clear description of machining characteristics of metals and the esablishment of new angle relationships.

(2) Lee-Shaffer solution is a special solution of the derived equation when m is l.

(3) Shear angle ϕ is independent of the undeformed chip thickness and the accordance between theory and experiment quoted in data by Egleston(1959) is well.

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